THE SOLUTION OF PROBLEMS DEALING WITH CONVECTIVE HEAT TRANSFER IN TUBES BY THE BUBNOV-GALERKIN METHOD

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An analytical method is developed for the solution of internal problems of nonsteady heat transfer for the laminar flow of a fluid through tubes exhibiting various perpendicular cross sections. The method is based on the combined application of integral transformations and variational calculus. Transient processes are studied for the nonsteady heat transfer in circular tubes and in plane-parallel channels, when the temperature at the inlet varies according to a specified law.

Particularly for purposes of automatic control, contemporary engineering procedures call for the calculation of nonsteady heat transfer in the flow of fluids through tubes (channels) of diverse geometric shapes in perpendicular cross section. With the development of atomic energy, increasing attention is currently being devoted to the study of the transient processes in the nonsteady regime found in heat exchangers.

The author of [4] describes the theoretical investigation of the nonsteady problems of convective energy transfer within a circular tube and in a plane-parallel channel for a laminar hydrodynamically stabilized flow of an incompressible fluid for the case in which the dissipation of energy and the axial heat conduction are neglected in the energy equations. The problems are solved for constant temperature regimes at both the inlet and at the walls of the tube.

In this paper we will deal with the more general three-dimensional problem of the liminar flow of an incompressible fluid in tubes of arbitrary cross section, when the temperature of the fluid at the inlet varies with time according to a specified law, and when the temperature of the wall varies along the length of the tube. The boundary-value problem is initially solved in general form, and then we consider a number of special problems for specific temperature regimes at the inlet and at the walls. Analytically, the method is based on the combined application of two contemporary apparatuses of applied mathematics—integral transformations and variational methods. This method exhibits a number of advantages relative to the other analytical methods of solving the internal problems of convective heat transfer which are known in the literature.

In [3] this method is used to solve the internal problems of convective heat transfer for the steady-state regime.

The energy equation for an incompressible flow in a tube with a lateral cross section D, and also the initial and boundary conditions for certain assumptions, are written in the form

$$\frac{\partial T}{\partial t} + w(x, y)\frac{\partial T}{\partial z} = a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\eta}{c\gamma}\,\text{Diss.Fkt}\left(\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}\right), \tag{1}$$

$$[T(x, y, z, t)]_{t=0} = f(x, y, z) = T_0 = \text{const}, \tag{2}$$

$$[T(x, y, z, t)]_{z=0} = \varphi(t),$$

$$[T(x, y, z, t)]_{\Gamma} = \varphi_w(z, t)$$

$$(x, y \in D,\quad 0 \leqslant z < \infty), \tag{3}$$

where Γ is the side surface of a cylindrical tube whose generatrix is parallel to the z-axis.

Let

$$\bar{T}^*(x, y, s, p) = ps\int_0^\infty\int_0^\infty T(x, y, z, t)\exp[-(pt + sz)]\,dt\,dz. \tag{4}$$

After applying a double Laplace transform with respect to time t and the coordinate z to Eq. (1), taking into consideration conditions (2) and (3), we obtain

$$a\left(\frac{\partial^2 \bar{T}^*}{\partial x^2} + \frac{\partial^2 \bar{T}^*}{\partial y^2}\right) - [p + sw(x, y)]\,\bar{T}^*(x, y, s, p) + \bar{\theta}^*(x, y, s, p) = 0, \tag{5}$$

where

$$\bar{\theta}^*(x, y, s, p) = pT_0 + sw(x, y)\,\bar{\varphi}(p) + \frac{\eta}{c\gamma}\,\text{Diss.Fkt}.$$

For the transformed energy equation (5) we obtain the following boundary conditions in the region of La-

Values of the Coefficients β_k, γ_k, and α_k and of the Functions $\varphi_k^*(\xi)$ and $\varphi_k^{**}(\xi)$

k	β_k	γ_k	α_k	$\varphi_k^*(\xi)$	$\varphi_k^{**}(\xi)$
1	5.782	3.314	0.791	$1.601-2.310\xi^2+0.818\xi^4-0.109\xi^6$	$1.464-2.612\xi^2+1.591\xi^4-0.44\xi^6$
2	30.718	47.878	0.644	$-1.016-6.810\xi^2-9.794\xi^4+4.00\xi^6$	$-0.707+5.999\xi^2+9.588\xi^4+4.282\xi^6$
3	113.494	322.410	0.321	$0.653-8.273\xi^2+20.263\xi^4-12.643\xi^6$	$0.335-5.117\xi^2+8.801\xi^4-4.019\xi^6$

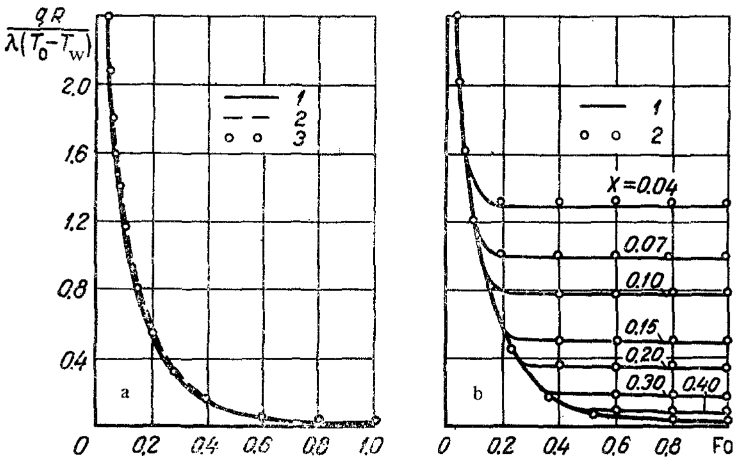

Fig. 1. Comparison of heat fluxes at unsteady-state heat transfer with unsteady-state thermal conductivity (a) and change in unsteady heat flux on circular tube surface under transient conditions (b): a) 1-results of present study; 2-solution for thermal conductivity; 3-according to data of [4]; b) 1-according to formula (25); 2-according to [4].

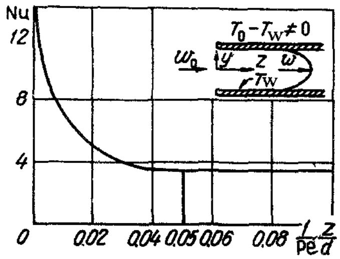

Fig. 2. Change in local Nusselt number in circular tube.

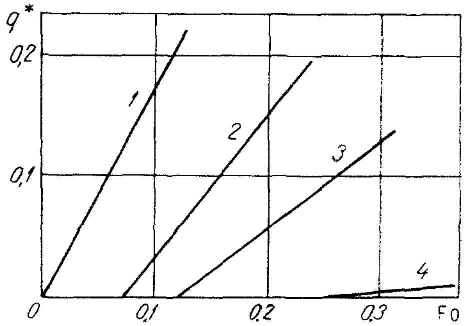

Fig. 3. Change in heat flux at the wall for a linear temperature variation at the inlet.

place transforms:

$$[\overline{T}^*(x,\ y,\ s,\ p)]_\Gamma = \overline{\varphi}^*_w(s,\ p). \quad (6)$$

We will seek the solution of the boundary-value problem (5) and (6) with the Bubnov-Galerkin variational

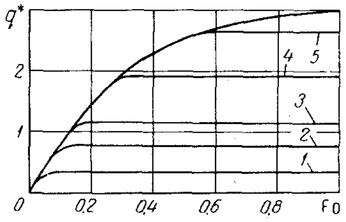

Fig. 4. Heat transfer from slot channel walls in the case of energy dissipation: 1) X = 0.005; 2) 0.1; 3) 0.2; 4) 0.4; 5) 0.8.

method [1]. Let us assume that we have selected a system of coordinate functions

$$\psi_1(x,\ y),\quad \psi_2(x,\ y),\ \ldots,\ \psi_n(x,\ y), \quad (7)$$

satisfying zero boundary conditions, i.e.,

$$[\psi_k(x,\ y)]_\Gamma = 0 \quad (k = 1,\ 2,\ \ldots,\ n).$$

The approximate value of the function $\overline{T}^*(x, y, s, p)$ is determined in a family of functions of the form

$$\overline{T}^*_n(x,\ y,\ s,\ p) = \overline{\varphi}^*_{\text{cr}}(s,\ p) + \sum_{k=1}^{n} \overline{a}^*_k(s,\ p)\,\psi_k(x,\ y). \quad (8)$$

Let us find the discrepancy for Eq. (5) when $\overline{T}^* = \overline{T}^*_n(x, y, s, p)$,

$$\varepsilon_n[\overline{a}^*_1(s,\ p),\ \ldots,\ \overline{a}^*_n(s,\ p),\ x,\ y] =$$

$$= a\left(\frac{\partial^2 \overline{T}^*_n}{\partial x^2} + \frac{\partial^2 \overline{T}^*_n}{\partial y^2}\right) - [p + sw(x,\ y)]\,\overline{T}^*_n + \overline{\theta}^*, \quad (9)$$

which is different from zero in region D.

The coefficients $\overline{a}^*_k(s,p)$ for which discrepancy (9) exhibits the least deviation from zero for all values of x and y from the region D, according to the Bubnov-Galerkin method, is determined from the following system [1]:

$$\iint_D \varepsilon_n[\overline{a}^*_1(s,\ p),\ \overline{a}^*_2(s,\ p),\ \ldots,\ \overline{a}^*_n(s,\ p),\ x,\ y] \times$$

$$\times\, \psi_m(x,\ y)\,dxdy = 0$$

or

$$\sum_{k=1}^{n} (A_{mk} + B_{mk}p + C_{mk}s)\,\overline{a}^*_k(s,\ p) = D_m$$

$$(m = 1,\ 2,\ \ldots,\ n), \quad (10)$$

where

$$A_{mk} = -a\iint_D \left(\frac{\partial^2 \psi_k}{\partial x^2} + \frac{\partial^2 \psi_k}{\partial y^2}\right)\psi_m(x,\ y)\,dxdy,$$

$$\iint_D \psi_m\psi_k\,dxdy = B_{mk},$$

$$C_{mk} = \iint_D w(x,\ y)\,\psi_k\psi_m\,dxdy,$$

$$D_m = \iint_D \overline{\theta}^*(x,\ y,\ s,\ p)\,\psi_m\,dxdy. \quad (11)$$

Having determined the coefficients $\overline{a}^*_k(s,p)$ from system (10) and crossing over to the preimage region from images (8), we obtain a solution for the basic problem in the form

$$T_n(x,\ y,\ z,\ t) = \varphi_w(z,\ t) + \sum_{k=1}^{n} a_k(z,\ t)\,\psi_k(x,\ y). \quad (12)$$

This represents the formal description of the combined application of the double integral Laplace transforms and the variational methods to the problems of convective heat transfer in nonsteady regimes. It should be pointed out that the stabilized field of velocities w(x, y) is assumed to be known.

Let us consider certain special problems of heat transfer.

1. Heat transfer in a circular tube.

If we neglect the function of the dissipation of the heat of friction, the energy equation has the form

$$\frac{\partial T}{\partial t} + w(r)\frac{\partial T}{\partial z} = \frac{a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right). \quad (13)$$

Substituting the Hagen-Poiseuille equation

$$w(r) = 2w_0\left[1 - \left(\frac{r}{R}\right)^2\right]$$

and introducing the dimensionless variables

$$\tau = \text{Fo} = \frac{t\nu}{R^2\,\text{Pr}},\quad X = \frac{1}{\text{Re}\,\text{Pr}}\,\frac{z}{R},\quad \xi = \frac{r}{R},$$

we obtain

$$\frac{\partial T}{\partial \tau} + (1 - \xi^2)\frac{\partial T}{\partial X} = \frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi\frac{\partial T}{\partial \xi}\right). \quad (14)$$

Let us study heat transfer for constant boundary conditions:

$$[T(\xi,\ X,\ \tau)]_{\tau=0} = T_0,$$

$$[T(\xi,\ X,\ \tau)]_{X=0} = T_0, \quad (15)$$

$$[T(\xi,\ X,\ \tau)]_{\xi=1} = T_w,\quad \left(\frac{\partial T}{\partial \xi}\right)_{\xi=0} = 0. \quad (16)$$

The boundary problem (14)–(16) in the Laplace-transform region is brought to the form

$$\frac{d}{d\xi}\left(\xi\frac{d\overline{T}^*}{d\xi}\right) - [p + (1 - \xi^2)s]\,\xi\,\overline{T}^*(\xi,\ s,\ p) +$$

$$+ T_0[p + (1 - \xi^2)s] = 0, \quad (17)$$

$$[\bar{T}^*(\xi, s, p)]_{\xi=1} = T_w, \quad \left(\frac{\partial \bar{T}^*}{\partial \xi}\right)_{\xi=0} = 0. \tag{18}$$

We will seek the approximate value of $\bar{T}^*(\xi, s, p)$ in a family of a linear composition of the form

$$\bar{T}_n^*(s, \xi, p) = T_w + \sum_{k=1}^{n} \bar{a}_k^*(s, p)(1-\xi^2)\xi^{2(k-1)}. \tag{19}$$

To reduce the amount of mathematical calculation, we present the solution in first approximation. With n = = 1 the discrepancy is equal to

$$\varepsilon_1[\bar{a}_1^*(s, p), \xi] = \{-4\xi - p(1-\xi^2)\xi - s(1-\xi^2)^2\xi\} \times$$

$$\times \bar{a}_1^*(s, p) + (T_0 - T_w)[p + (1-\xi^2)s]\xi \neq 0.$$

Requiring the discrepancy ε_1 to be orthogonal with respect to the coordinate function $\psi_1(\xi) = 1 - \xi^2$, we obtain

$$\bar{a}_1^*(s, p) = \frac{(T_0 - T_w)(D_1 p + D_2 s)}{A + Bp + Cs}, \tag{20}$$

where

$$A = 1, \quad B = \frac{1}{6},$$

$$C = \frac{1}{8}, \quad D_1 = \frac{1}{4}, \quad D_2 = \frac{1}{6}.$$

From the operational-calculus table [2] we have

$$\frac{p}{A + Bp + Cs} \risingdotseq$$

$$\risingdotseq \begin{cases} \frac{1}{B}\exp\left(-\frac{A}{B}\tau\right) & \text{when } X > \frac{C}{B}\tau, \\ 0 & \text{when } X < \frac{C}{B}\tau. \end{cases} \tag{21}$$

Consequently,

$$\bar{a}_1^*(s, p) \risingdotseq a_1(X, \mathrm{Fo}) =$$

$$= \begin{cases} \frac{D_1}{B}\exp\left(-\frac{A}{B}\mathrm{Fo}\right) & \text{when } X > \frac{C}{B}\mathrm{Fo}, \\ \frac{D_2}{B}\exp\left(-\frac{A}{C}X\right) & \text{when } X < \frac{C}{B}\mathrm{Fo}. \end{cases} \tag{22}$$

In first approximation, the temperature in the fluid flow is found in the form

$$T(\xi, X, \mathrm{Fo}) =$$

$$= \begin{cases} T_w + \frac{3}{2}\exp(-6\,\mathrm{Fo})(1-\xi^2) \\ \quad \text{when } X > 0.75\,\mathrm{Fo}, \\ T_w + \frac{4}{3}\exp(-8X)(1-\xi^2) \\ \quad \text{when } X < 0.75\,\mathrm{Fo}. \end{cases} \tag{23}$$

Dropping a number of intervening calculations, we write the temperature field in third approximation:

$$\frac{T(\xi, X, \mathrm{Fo}) - T_w}{T_0 - T_w} =$$

$$= \begin{cases} \sum_{k=1}^{3} \varphi_k^*(\xi)\exp(-\beta_k \mathrm{Fo}) & \text{when } X > \alpha_k \mathrm{Fo}, \\ \sum_{k=1}^{3} \varphi_k^{**}(\xi)\exp(-\gamma_k X) & \text{when } X < \alpha_k \mathrm{Fo}. \end{cases} \tag{24}$$

The values of the coefficients β_k, γ_k, and α_k and of the functions $\varphi_k^*(\xi)$ and $\varphi_k^{**}(\xi)$ are presented in the table.

The heat flow q needed to maintain a constant wall temperature beyond the jump in T_w ($\Delta T = T_0 - T_w \neq \neq 0$) is found from solution (24) according to the following formula:

$$\frac{qR}{\lambda(T_0 - T_w)} = -\sum_{k=1}^{3} \begin{cases} \left(\frac{\partial \varphi_k^*}{\partial \xi}\right)_{\xi=1}\exp(-\beta_k \mathrm{Fo}) \\ \quad \text{when } X > \alpha_k \mathrm{Fo}, \\ \left(\frac{\partial \varphi_k^{**}}{\partial \xi}\right)_{\xi=1}\exp(-\gamma_k X) \\ \quad \text{when } X < \alpha_k \mathrm{Fo}. \end{cases} \tag{25}$$

Let us analyze the process of heat transfer immediately after the jump in the wall temperature ($T_w - - T_0 \neq 0$, $\mathrm{Fo} > 0$). In that part of the tube which has not yet been reached by the fluid which, prior to the jump, was outside of the tube, the temperature conditions at the inlet have no effect on the heat-transfer process. In the case of a uniform wall temperature, for this region there is no change in heat transfer over the length of the tube, and the conduction term in Eq. (13) is equal to zero ($w(\partial T/\partial z) = 0$). Thus the problem is reduced to the solution of an equation of nonsteady heat conduction for a solid unbounded cylinder. The heat flow in these segments is determined from the top line in (25) and yields the following:

$$\frac{qR}{\lambda(T_0 - T_w)} =$$

$$= 2.002\exp(-5.782\,\mathrm{Fo}) + 1.556\exp(-30.718\,\mathrm{Fo}) +$$

$$+ 11.352\exp(-113.494\,\mathrm{Fo}). \tag{26}$$

Comparison of relationship (26) with the exact solution of nonsteady heat conduction and the results obtained in the investigation carried out by the author of [4] are shown in Fig. 1a. Figure 1b shows the curves plotted according to Eq. (25) for various values of x, and these are compared with the data of [4], and from these we see that they are in good agreement.

The temperature in the fluid flow, given a sufficient amount of time ($\mathrm{Fo} > \alpha_k^{-1}X$) in third approximation is written in the form

$$\frac{T(\xi, X, \mathrm{Fo}) - T_w}{T_0 - T_w} =$$

$$= \sum_{k=1}^{3} \varphi_k^{**}(\xi)\exp(-\gamma_k X) \quad \left(X = \frac{1}{\mathrm{RePr}}\frac{z}{R}\right). \tag{27}$$

The local Nusselt number referred to the mean-integral temperature with respect to the flow rate of the incompressible fluid is expressed by means of the relationship

$$\mathrm{Nu} = N(z) = \left[2.992 + 1.260\exp\left(-\frac{20.282}{\mathrm{Pe}}\frac{z}{R}\right) - 1.796\exp\left(-\frac{157.548}{\mathrm{Pe}}\frac{z}{R}\right)\right] \times \left[0.816 - 0.124\exp\left(-\frac{20.282}{\mathrm{Pe}}\frac{z}{R}\right) - 0.306\exp\left(-\frac{157.548}{\mathrm{Pe}}\frac{z}{R}\right)\right]^{-1}, \quad (28)$$

from which the limit value of $\mathrm{Nu}_{\mathrm{stab}} = 3.66$. This quantity differs from the familiar Nusselt number (3.65) by only 0.05%. The local Nusselt number N(z), even at a distance of

$$\frac{a}{w_0 d}\frac{z}{d} = 0.05,\ d = 2R \quad (29)$$

beyond the inlet to the tube (for fluids flowing from a reservoir) differs from 3.66 by no more than 1% (see Fig. 2).

Let us determine the temperature in the flow of an incompressible fluid when the temperature at the inlet to the tube varies according to a linear law. For this we have to solve Eq. (14) for the following boundary conditions:

$$[T(\xi, X, \mathrm{Fo})]_{\mathrm{Fo}=0} = T_0,$$

$$[T(\xi, X, \mathrm{Fo})]_{X=0} = \varphi(t) = T_0 + \Delta T t = T_0(1 + \mathrm{Pd}\mathrm{Fo}), \quad (30)$$

$$\mathrm{Pd} = \frac{\Delta T R^2}{a T_0},\quad \mathrm{Fo} = \frac{at}{R^2}. \quad (31)$$

Equation (14) in the Laplace-transform region is brought to the form

$$\frac{d}{d\xi}\left(\xi\frac{d\bar{T}^*}{d\xi}\right) - [p + (1-\xi^2)s]\,\xi\,\bar{T}^*(\xi, s, p) + T_0[p + (1-\xi^2)s]\,\xi + \frac{\xi(1-\xi^2)s}{p}\mathrm{Pd} = 0. \quad (32)$$

Having determined the coefficient $\bar{a}_1^*(s,p)$ from system (10) when n = 1 and crossing over to the preimage region, we find the solution in first approximation in the following form:

$$\frac{T(\xi, X, \mathrm{Fo}) - T_0}{T_0\mathrm{Pd}} = \begin{cases} \left(\mathrm{Fo} - \frac{4}{3}X\right)\exp(-8X)(1-\xi^2) \\ \text{when } \mathrm{Fo} > \frac{4}{3}X, \\ 0 \text{ when } \mathrm{Fo} < \frac{4}{3}X. \end{cases} \quad (33)$$

The heat flow q needed to maintain the initial wall temperature (T_0) is found to be the following:

$$q^* = \frac{qR}{\lambda\,\mathrm{Pd}} = \begin{cases} \left(2\mathrm{Fo} - \frac{8}{3}X\right)\exp(-8X) \\ \text{when } \mathrm{Fo} > \frac{4}{3}X, \\ 0 \text{ when } \mathrm{Fo} < \frac{4}{3}X = \frac{4}{3\mathrm{RePr}}\frac{z}{R}. \end{cases} \quad (34)$$

The time variation of the local heat flow at various points x is shown in Fig. 3.

While the temperature of the fluid entering the tube varies with time according to a periodic function, i. e.,

$$[T(\xi, X, \mathrm{Fo})]_{X=0} = T_0 + \Delta T\sin(\mathrm{Pd}\mathrm{Fo}), \quad (35)$$

where $\mathrm{Pd} = \omega R^2/a$ is the Predvoditelev number, the temperature within the tube can be calculated from the approximate formula

$$\frac{T(\xi, X, \mathrm{Fo}) - T_0}{\Delta T} = \begin{cases} \frac{4}{3}\exp\left(\frac{-8}{\mathrm{RePr}}\frac{z}{R}\right) \times \sin\left[\mathrm{Pd}\left(\mathrm{Fo} - \frac{4}{3\mathrm{RePr}}\frac{z}{R}\right)\right]\left[1 - \left(\frac{r}{R}\right)^2\right] \\ \text{when } \mathrm{Fo} > \frac{4}{3}\frac{1}{\mathrm{RePr}}\frac{z}{R} = \frac{4}{3}X, \\ 0 \text{ when } \mathrm{Fo} < \frac{4}{3}\frac{1}{\mathrm{RePr}}\frac{z}{R}; \end{cases} \quad (36)$$

whence

$$q^* = \frac{qR}{\lambda\Delta T} = \frac{8}{3}\sin\left[\mathrm{Pd}\left(\mathrm{Fo} - \frac{4}{3}\frac{1}{\mathrm{RePr}}\frac{z}{R}\right)\right] \times \exp\left(-\frac{8}{\mathrm{PrRe}}\frac{z}{R}\right). \quad (37)$$

2. A plane-parallel channel.

The boundary-value problem (1)–(3) is written out in the following equations:

$$\frac{\partial T}{\partial t} + w(y)\frac{\partial T}{\partial z} = a\frac{\partial^2 T}{\partial y^2} + \frac{\eta}{c\gamma}\left(\frac{\partial w}{\partial y}\right)^2, \quad (38)$$

$$w(y) = \frac{3w_0}{2}\left[1 - \left(\frac{y}{b}\right)^2\right],$$

$$[T(y, z, t)]_{t=0} = T_0 = \mathrm{const}$$

$$(-b \leqslant y \leqslant b,\ 0 \leqslant z < \infty), \quad (39)$$

$$[T(y, z, t)]_{z=0} = \varphi_0(t),\quad [T(y, z, t)]_{y=(-1)^k b} = \varphi_k(z, t)$$

$$(k = 1, 2). \quad (40)$$

In the Laplace-transform region ($T(y, z, t) \doteqdot T^*(y, s, p)$)

we have

$$a\frac{d^2\bar{T}^*}{dy^2}-[p+w(y)s]\bar{T}^*$$

$$+[pT_0+w(y)s\bar{\varphi}_0^*(p)]+\frac{\eta}{c\gamma}\left(\frac{\partial w}{\partial y}\right)^2=0, \qquad (41)$$

$$[\bar{T}^*(y,s,p)]_{y=(-1)^k b}=\bar{\varphi}_k^*(s,p) \qquad k=1,2). \qquad (42)$$

The solution of the boundary-value problem will be sought in the family of functions

$$\bar{T}_n^*(y,s,p)=\frac{1}{2}\left\{\frac{y}{b}[\bar{\varphi}_2^*(s,p)-\bar{\varphi}_1^*(s,p)]+\right.$$

$$\left.+[\varphi_2^*(s,p)+\bar{\varphi}_1^*(s,p)]\right\}+$$

$$+\sum_{k=1}^{n}\bar{a}_k^*(s,p)\left[1-\left(\frac{y}{b}\right)^4\right]\left(\frac{y}{b}\right)^{2(k-1)}. \qquad (43)$$

In all other respects, the problem is solved in a manner similar to the previous problem.

The heat transfer is governed exclusively by the heat of friction.

Let

$$\varphi_1(z,t)=\varphi_2(z,t)=T_0=\text{const},\ \varphi_0(t)=T_0, \qquad (44)$$

i. e., the temperature of the channel wall is equal to the temperature of the liquid (gas) at the inlet. It is evident that in this case the variation in the temperature of the fluid flow and the removal of heat through the channel walls are due entirely to the heat of friction.

We will continue the solution of the problem for the first approximation, i. e., we will seek the temperature field within the transform region among functions of the form

$$\bar{T}^*(y,s,p)=T_0+\bar{a}_1^*(s,p)\left[1-\left(\frac{y}{b}\right)^4\right]. \qquad (45)$$

Let us substitute the value of (45) into Eq. (41), under the condition that (44) is uniquely defined; in this case

$$\varepsilon_1[y,\bar{a}_1^*(s,p)]=\left\{-\frac{12a}{b^2}\left(\frac{y}{b}\right)^2-\right.$$

$$\left.-[p+w(y)s]\left[1-\left(\frac{y}{b}\right)^4\right]\right\}\bar{a}_1^*(s,p)+$$

$$+\frac{9\eta w_0^2}{c\gamma b^2}\left(\frac{y}{b}\right)^2\neq 0.$$

Requiring the discrepancy ε_1 to be orthogonal with respect to the coordinate function $\psi_1(y)=[1-(y/b)^4]$ in the region D $\{-b\le y\le b\}$, we have

$$\bar{a}_1^*(s,p)=\frac{D}{A+Bp+Cs},$$

where

$$A=\frac{32a}{7},\quad B=\frac{64b}{45},$$

$$C=1.71\,w_0 b,\ D=\frac{24w_0}{7b}\frac{\eta}{c\gamma}.$$

From the operational-calculus tables [2] we find that

$$\bar{a}_1^*(s,p)\doteqdot a_1(z,t)=$$

$$=\begin{cases}\dfrac{3}{4}\dfrac{\eta w_0^2}{\lambda}[1-\exp(-3.214\,\text{Fo})] \\ \qquad\text{for } z>1,2\,w_0t, \\ \dfrac{3}{4}\dfrac{\eta w_0^2}{\lambda}\left[1-\exp\left(-\dfrac{2.673}{\text{Pe}}\dfrac{z}{b}\right)\right] \\ \qquad\text{for } z<1,2\,w_0t,\end{cases}$$

where

$$\text{Fo}=\frac{at}{b^2},\quad \text{Pe}=\text{Re}\,\text{Pr}=\frac{w_0 b}{a}.$$

The variation of temperature within the plane-parallel channel on dissipation of energy and equality of the wall temperature to the initial fluid temperature is written, in approximate terms, in the following two analytical expressions:

$$T(y,z,t)=$$

$$=T_0+\frac{3}{4}\frac{\eta w_0^2}{\lambda}[1-\exp(-3.214\,\text{Fo})]\left[1-\left(\frac{y}{b}\right)^4\right]$$

$$\text{when } z>1.2\,w_0t, \qquad (46)$$

$$T(y,z,t)=T_0+$$

$$+\frac{3}{4}\frac{\eta w_0^2}{\lambda}\left[1-\exp\left(-\frac{2.673}{\text{Pe}}\frac{z}{b}\right)\right]\left[1-\left(\frac{y}{b}\right)^4\right]$$

$$\text{when } z<1.2\,w_0t. \qquad (47)$$

The temperature at that segment of the channel which has not yet been reached by the fluid which was outside the channel at t = 0 is determined from formula (46). At these points the heat-transfer process is unaffected by the inlet conditions and everything proceeds as in the case of an unbounded slotted channel—when the wall temperature is uniform along the length of the channel—in which case the convection term in Eq. (38) is equal to zero. We then derive the solution for the problem of nonsteady heat conduction for an unbounded plate with internal sources whose local parameters are parabolically distributed over the entire depth of the plate.

Formula (47) is used to determine the temperature for the rather long period of time t in which the original fluid within the channel is expelled by the fluid from the reservoir. This solution, beginning from the point

$$\frac{1}{\text{Pe}}\frac{z}{d}=0.5\ (d=2b) \qquad (48)$$

and beyond differs from the expression

$$T_{\text{stab}}(y) = T_0 + \frac{3}{4}\frac{\eta w_0^2}{\lambda} \times \left[1 - \left(\frac{y}{b}\right)^4\right] \qquad (49)$$

by no more than 4%. The temperature distribution at the stabilized segment is thus determined from relationship (49). This expression is in exact agreement with the familiar Schlichting solution [5].

Equation (48) determines the length of the thermal-stabilization segment for the case of energy dissipation.

Differentiating (46) and (47), we have

$$q^* = \frac{qB}{\eta w_0^2} =$$

$$= \begin{cases} 3\left[1 - \exp(-3.214\,\mathrm{Fo})\right], & z > 1.2\, w_0 t, \\ 3\left[1 - \exp\left(-\frac{2.673}{\mathrm{Pe}}\frac{z}{b}\right)\right] & z < 1.2\, w_0 t. \end{cases} \qquad (50)$$

Figure 4 shows the curves for the change in the heat flow q^* for any instant of time at various points $X = (1/\mathrm{Pe})(z/b)$.

In conclusion, we will point out that the proposed method of calculating the interior problems of convective heat transfer in the nonsteady regime makes it possible to resolve a number of new problems pertaining to tubes and channels with "nonclassical" profiles of perpendicular cross section. In particular, for tubes of elliptical cross section and of a cross section in the shape of an equilateral triangle we can solve a number of problems concerned with the practical aspects of convective heat transfer in the case of laminar fluid flow. The stabilized velocity field of the fluid flow should be represented in this case by the familiar hydrodynamic equations

$$w(x, y) = 2 w_0 \left[1 - \left(\frac{x}{a}\right)^2 \left(\frac{y}{b}\right)^2\right],$$

$$w(x, y) =$$

$$= 15 w_0 \left[\left(\frac{y}{\sqrt{3}h}\right)^2 - \left(\frac{x}{h}\right)^2\right]\left[1 - \left(\frac{y}{\sqrt{3}h}\right)\right],$$

where w_0 is the average velocity; 2h is the side of the triangle. Similar problems will be considered in another paper.

NOTATION

D is the cross-section of the cylindrical tube; p and s are the parameters of the double Laplace transform in the Laplace-transform region; $\overline{T}^*(x, y, s, p)$ is the temperature field in the fluid flow after Laplace transformation; is the sign of transition from image to preimage and back; w(x, y) is the stabilized velocity field of the fluid flow; w_0 is the mean velocity; ν is the kinematic viscosity; η is the dynamic viscosity; λ and a are the thermal conductivity and the thermal diffusivity coefficients; τ is the dimensionless time (the Fo number); X is the dimensionless coordinate along the tube axis.

REFERENCES

1. S. G. Mikhlin, Variational Methods in Mathematical Physics [in Russian], Gostekhizdat, 1957.
2. V. A. Ditkin and A. P. Prudnikov, The Operational Calculus in Two Variables and its Application [in Russian], Fizmatgiz, 1958.
3. P. V. Tsoi, Teploenergetika, no. 8, 1967.
4. R. Siegel, Trans. ASME, APMW-21, 1, 1959.
5. H. Schlichting, Zs. angew. Math. Mech., 31, 78, 1951.

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